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Distributed Piezoelectric Element Method for Vibration Control of Smart Plates

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A new method for active control of plates is presented that includes methods for designing piezoelectric modal sensors and modal actuators. The piezoelectric modal sensor and modal actuator are designed by cutting all of the piezoelectric layers into several independent elements. The modal coordinates and the modal velocities can be obtained from the charges and currents generated by the sensor elements, and the modal actuator is designed by modulating the spatial distribution of voltage applied on the piezoelectric actuator. A scheme for modal control of smart plates is given. Moreover, improvement of the controllabilty of the plates due to segmentation actuator lamina is discussed.

I. Introduction

SMART structures for vibration control of flexible space structures have attracted a considerable amount of research in recent years. A smart structure (or intelligent structure) used in vibration control can be viewed as a structure or structure component with distributed sensors and actuators that can sense the excitations applied by its environment and can also act properly through control devices to compensate for undesired effects or to enhance desired effects.[1] The distributed sensors and actuators in such structures are usually made of piezoelectric materials such as lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF).

Application of smart structures to vibration control may be traced to Bailey and Hubbard,[2] who used PVDF as the distributed actuator in a cantilever beam to control its vibration. In the following 10 years, the models, basic equations, control laws, finite element analysis methods, and experiments for smart structures were investigated by many researchers.[3–12]

The independent modal space control (IMSC)[13] method is often used in vibration control that requires the distributed sensor/actuator to sense/actuate desired modes. Lee and Moon[14] and Lee et al.[15] presented a method to design modal sensors/actuators by means of modulating the shape together with varying the polarization of piezoelectric layers. Consequently, they achieved critical damping of the first mode of a beam. However, when the number of the controlled modes varies, or the parameters such as the density or the thickness of the structure vary, as a part of the structure, the sensor layer and the actuator layer must be reshaped. We present a method called the distributed piezoelectric element (DPE) method, which uses segmented piezoelectric actuators and sensors, that includes a new method for designing piezoelectric modal sensors and modal actuators. In this method, the piezoelectric sensor/actuator layers are cut into several independent pieces called sensor/actuator elements that independently sense/actuate a local strain state. The modal coordinates and the modal velocities can be obtained from the charges and currents generated by the sensor elements, and the modal actuator is designed by modulating the spatial distribution of voltage applied on the piezoelectric actuator. Based on the modal sensor and modal actuator, the modal control of the smart plate is performed. Finally, a simulation example is given to demonstrate the presented method.

II. Basic Equations of a Smart Plate

Consider a thin elastic plate: Two lateral isotropic piezoelectric laminas are bonded onto the upper surface of the plate as an actuator and on the lower surface of the plate as a sensor, respectively. For the sensor lamina, host plate, and actuator lamina, Young's moduli are Y_1, Y_2, and Y_3; Poisson's ratios are μ_1, μ_2, and μ_3; and the mass densities are ρ_1, ρ_2, and ρ_3, respectively. The z coordinates of the lower face and the upper face of the sensor lamina are z_0 and z_1, respectively; those of the actuator lamina are z_2 and z_3, respectively, as shown in Fig. 1. The tensile axes of the piezoelectric laminas coincide with the axes of the plate considered.

The charge output of the sensor lamina may be derived as

$$q(t) = -e_{31}^{s} \iint_S r_s \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \mathrm{d}x\, \mathrm{d}y \tag{1}$$

where S is the area covered by the piezoelectric sensor layer, $r_s = \frac{1}{2}(z_0 + z_1)$ is the z coordinate of the midplane of sensor lamina, and e_{31}^{s} is the piezoelectric stress constant of the sensor lamina. Equation (1) establishes the relationship between the charge output and the strain of the plate, and so it is called the sensor equation.

By the differentiation of Eq. (1) with respect to t, we have

$$I(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t} = -e_{31}^{s} \iint_S r_s \left(\frac{\partial^3 w}{\partial x^2 \partial t} + \frac{\partial^3 w}{\partial y^2 \partial t} \right) \mathrm{d}x\, \mathrm{d}y \tag{2}$$

which describes the relationship between the output electric current and the even strain rate of the sensor lamina.

The differential equation of motion of the smart plate may be derived as

$$\rho h \frac{\partial^2 w}{\partial t^2} + D\nabla^4 w = -e_{31}^{a} r_a \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) \tag{3}$$

where

$$\rho h = \sum_{i=1}^{3} \rho_i (z_i - z_{i-1})$$

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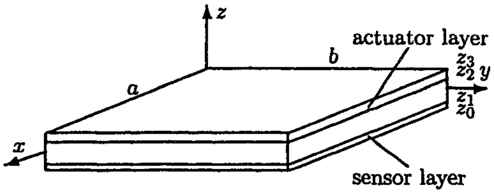

Fig. 1 Smart plate with piezoelectric sensor and actuator layers.

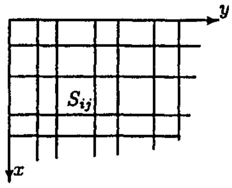

Fig. 2 Segmentation of the sensor layer.

is the equivalent mass density of the smart plate and

$$D = \frac{1}{3}\sum_{i=1}^{3}\frac{Y_i}{1-\mu_i^2}\left(z_i^3 - z_{i-1}^3\right) \tag{4}$$

is the bending stiffness of the plate, e_{31}^a is the piezoelectric stress constant of the actuator lamina, r_a is the z coordinate of the midplane of the actuator lamina, and $V(x, y, t)$ is the voltage applied on the actuator lamina in the thickness direction. Equation (3) is an equation coupled with mechanical and electric variables that describes the relationship between the vibration of smart plate and the excitation voltage, and so it is also called the piezoelectric actuator equation.

III. Modal Sensor Design

The modal coordinates and modal velocities must be observed when using the modal control method to control the plate's vibration. Because the output obtained from Eqs. (1) and (2) is information of the averaged strain and the averaged strain rate of the plate, it is not useful in obtaining information concerning the strain of the modes. To draw the modal coordinates and modal velocities from the voltage output of the sensor lamina, we develop a fully distributed piezoelectric segments strategy. The sensor lamina bonded on the surface of the plate is separated into MN sensor elements S_{ij}, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$, as shown in Fig. 2.

By the employment of Eq. (1), the charge of the ijth sensor element caused by the strain of the plate can be obtained:

$$q_{ij}(t) = e_{31}^s \iint_{S_{ij}} \varepsilon(x, y, t)\,\mathrm{d}x\,\mathrm{d}y$$

$$i = 1, 2, \ldots, M; \quad j = 1, 2, \ldots, N \tag{5}$$

where

$$\varepsilon(x, y, t) = -r_s\left[\frac{\partial^2 w(x, y, t)}{\partial x^2} + \frac{\partial^2 w(x, y, t)}{\partial y^2}\right] \tag{6}$$

is the strain of the sensor lamina. By expressing the transverse displacement of the plate as a linear superposition of the modes of the plate and by considering only the first MN, we have

$$w(x, y, t) = \sum_{m=1}^{M}\sum_{n=1}^{N}\eta_{mn}^*(t)W_{mn}(x, y) \tag{7}$$

where $W_{mn}(x, y)$ is the mnth mode shape of the plate and $\eta_{mn}^*(t)$ is the mnth observed mode coordinate. Substituting Eq. (7) into Eq. (5) yields

$$q_{ij}(t) = e_{31}^s \sum_{m=1}^{M}\sum_{n=1}^{N}\eta_{mn}^*(t)\iint_{S_{ij}}\mathcal{E}_{mn}(x, y)\,\mathrm{d}x\,\mathrm{d}y$$

$$i = 1, 2, \ldots, M; \quad j = 1, 2, \ldots, N \tag{8}$$

where

$$\varepsilon_{mn}(x, y) = -r_s\left[\frac{\partial^2 W_{mn}(x, y)}{\partial x^2} + \frac{\partial^2 W_{mn}(x, y)}{\partial y^2}\right] \tag{9}$$

is the mnth strain mode of sensor lamina. Equation (8) may be rewritten in matrix form

$$\{q(t)\} = e_{31}^s[\bar{\mathcal{E}}]\{\eta^*(t)\} \tag{10}$$

where $\{q(t)\} = (q_{11}, q_{12}, \ldots, q_{1N}, \ldots, q_{M1}, q_{M2}, \ldots, q_{MN})^T$, $\{\eta^*(t)\} = (\eta_{11}^*, \eta_{12}^*, \ldots, \eta_{1N}^*, \ldots, \eta_{M1}^*, \eta_{M2}^*, \ldots, \eta_{MN}^*)^T$, and $[\bar{\mathcal{E}}]$ is an $MN \times MN$ matrix concerning the integration of strain modes, whose elements are

$$\bar{\mathcal{E}}_{mn}(S_{ij}) = \iint_{S_{ij}}\mathcal{E}_{mn}(x, y)\,\mathrm{d}x\,\mathrm{d}y \tag{11}$$

The output charge of several neighboring elements can be superimposed together. For instance, the output charge superimposed from two neighboring elements is equivalent to combining these two elements into one. On the other hand, removing the output charge of an element is equivalent to removing this element. Therefore, the number, size, and location of sensor elements can be changed through superimposing and removing the output charge of sensor elements. Using this concept, $[\overline{\mathcal{E}^s}]$ can be made nonsingular by superimposing the output charge of elements. Without loss of generality, the matrix $[\overline{\mathcal{E}^s}]$ is assumed to be nonsingular. The MN mode coordinates of the plate can be solved from Eq. (10):

$$\{\eta^*(t)\} = \left(1/e_{31}^s\right)[\bar{\mathcal{E}}]^{-1}\{q(t)\} \tag{12}$$

If it is necessary to observe the mode velocities the electric current of piezoelectric sensor can be used as output. Employing Eq. (2) yields

$$\{\dot{\eta}^*(t)\} = \left(1/e_{31}^s\right)[\bar{\mathcal{E}}]^{-1}\{I(t)\} \tag{13}$$

where $\{I(t)\}$ is the vector that consists of the output currents of MN sensor elements.

Equations (12) and (13) show the process of mode observation performed by mode sensors; the observed values of former MN mode coordinates and mode velocities can be obtained from the output charge and current of all sensor elements.

IV. Modal Actuator Design

The modal actuator can actuate the designated modes without affecting other modes. We design the piezoelectric mode actuator by adjusting the spatial distribution of the voltage of the actuator lamina.

Transform the actuator equation (3) into modal equations:

$$\ddot{\eta}_{mn}(t) + \omega_{mn}^2\eta_{mn}(t) = -e_{31}^a r_a \iint_S\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)W_{mn}(x, y)\,\mathrm{d}x\,\mathrm{d}y$$

$$m, n = 1, 2, \ldots \tag{14}$$

where $\eta_{mn}(t)$ is the mnth mode coordinate and ω_{mn} is the mnth natural frequency.

It can be seen that only some specified modes can be actuated by the voltage applied to the piezoelectric actuator lamina through modulating the distribution, in space, of the voltage.

To this end, we design the voltage to be determined by

$$V(x, y, t) = \sum_{k=1}^{K}\sum_{l=1}^{L}p_{kl}(t)D\left[-\nabla^2 W_{kl}(x, y)\right]$$

$$= \sum_{k=1}^{K}\sum_{l=1}^{L}p_{kl}(t)\mathcal{M}_{kl}(x, y) \tag{15}$$

where K and L are the numbers of order of the required controlled modes $K \le M$ and $L \le N$ and

$$\mathcal{M}_{kl}(x, y) = D\left[-\nabla^2 W_{kl}(x, y)\right] \tag{16}$$

By inserting the voltage in Eq. (15) and employing the orthogonality of modes, we obtain

$$\ddot{\eta}_{kl}(t) + \omega_{kl}^2 \eta_{kl}(t) = e_{31}^a r_a \omega_{kl}^2 p_{kl}(t)$$
$$k = 1, 2, \ldots, K; \quad l = 1, 2, \ldots, L \quad (17)$$

Equation (17) shows that the former MN modes can be actuators, whereas the other modes are not actuated. Therefore, the piezoelectric modal actuators can be perfectly realized when the continuous distribution of the voltage in Eq. (15) is applied on the actuator lamina. However, such continuous distribution of the actuating voltage is hard to realize exactly.

To realize the distribution of the voltage in Eq. (15) practically, the piezoelectric actuator lamina is separated into QR elements $A_{11}, A_{12}, \ldots, A_{QR}$, and the continuous distribution, in space, of voltage $V(x, y, t)$ is approximated by piecewise constant voltage. The applied voltage to the ijth actuator element should be

$$V_{ij}(t) = \frac{1}{A_{ij}} \iint_{A_{ij}} V(x, y, t)\, \mathrm{d}x\, \mathrm{d}y$$
$$= \frac{1}{A_{ij}} \sum_{k=1}^{K} \sum_{l=1}^{L} p_{kl}(t) \iint_{A_{ij}} \mathcal{M}_{kl}(x, y)\, \mathrm{d}x\, \mathrm{d}y$$
$$i = 1, 2, \ldots, Q; \quad j = 1, 2, \ldots, R \quad (18)$$

Rewrite Eq. (18) in matrix form:

$$\{V(t)\} = [C][\bar{\mathcal{M}}]\{P(t)\} \quad (19)$$

where $\{V(t)\} = (V_{11}, V_{12}, \ldots, V_{1R}, \ldots, V_{Q1}, V_{Q2}, \ldots, V_{QR})^T$ is the vector that consists of QR voltages, $\{P(t)\} = (p_{11}, p_{12}, \ldots, p_{1L}, \ldots, p_{K1}, p_{K2}, \ldots, p_{KL})^T$ is the vector that consists of KL modal control forces, $[C] = \mathrm{diag}(1/A_{ij})$ is a $QR \times QR$ diagonal matrix, and $[\bar{\mathcal{M}}]$ is a $KL \times QR$ matrix, whose elements are

$$\bar{\mathcal{M}}_{kl}(A_{ij}) = \iint_{A_{ij}} \mathcal{M}_{kl}(x, y)\, \mathrm{d}x\, \mathrm{d}y \quad (20)$$

In this way, the required voltage on each piezoelectric actuator element can be determined from Eq. (18) or (19), and the modal control can be approximately realized by applying these voltages to actuator elements, respectively.

V. Modal Control

When the modal sensors and modal actuators are designed, the modal control of the smart plate can be performed by designing the modal control forces $p_{kl}(t)$ in the modal equation of motion (17). Employing negative feedback for modal coordinates and velocities, the $p_{kl}(t)$ are designed as

$$p_{kl}(t) = -g_{kl}\eta_{kl}^*(t) - h_{kl}\dot{\eta}_{kl}^*(t)$$
$$k = 1, 2, \ldots, K; \quad l = 1, 2, \ldots, L \quad (21)$$

where g_{kl} and h_{kl} are the mode control gains and $\eta_{kl}^*(t)$ and $\dot{\eta}_{kl}^*(t)$ are the coordinates and modal velocities observed by the modal sensor from Eq. (12) and (13), respectively. Therefore, the closed-loop modal equations become

$$\ddot{\eta}_{kl}(t) + h_{kl}e_{31}^a r_a \omega_{kl}^2 \dot{\eta}_{kl}^*(t) + \omega_{kl}^2 \eta_{kl}(t) + g_{kl}e_{31}^a r_a \omega_{kl}^2 \eta_{kl}^*(t) = 0$$
$$k = 1, 2, \ldots, K; \quad l = 1, 2, \ldots, L \quad (22)$$

The damping ratio of the klth mode of the controlled plate is approximately

$$\zeta_{kl}^c = \frac{h_{kl}e_{31}^a r_a \omega_{kl}}{2\sqrt{1 + g_{kl}e_{31}^a r_a}} \quad (23)$$

which can be used to select the control gains in Eq. (21).

Rewrite Eq. (21) in matrix form

$$\{P(t)\} = -[G]\{\eta^*(t)\} - [H]\{\dot{\eta}^*(t)\} \quad (24)$$

where $[G] = \mathrm{diag}(g_{kl})$ and $[H] = \mathrm{diag}(h_{kl})$ are diagonal modal gain matrices. By the substitution of Eq. (24) into Eq. (19), the control voltage distribution can be obtained:

$$\{V(t)\} = -[C][\bar{\mathcal{M}}][G]\{\eta^*(t)\} - [C][\bar{\mathcal{M}}][H]\{\dot{\eta}^*(t)\} \quad (25)$$

The independent modal space control can be performed by applying the obtained QR control voltages on the actuator elements, respectively.

Note that the observed modal coordinates and modal velocities are slightly different from the real ones because of the truncation of modes in Eq. (7). The observed modal coordinates and velocities are the sum of the real ones and the component of residual modes. Therefore, in practice, the low-pass filter is needed to remove the residual modes. If there are no residual modal components in the observed modes to be controlled, the stability of the residual modes are not affected when using the DPE method.

VI. Numerical Simulations

As an example, consider a simply supported uniform rectangular aluminum plate to which a PZT lamina and a PVDF lamina are bonded as actuator and sensor. The dimensions and parameters of the smart plate are given in Table 1.

The vibrations of the plate are excited by an impulse of 0.03 Ns at the point (25 cm, 13 cm) of the plate. Now, use the proposed method to control the plate's vibration. Let $M = 4$ and $N = 3$ so that the sensor lamina is separated equally into 12 rectangular sensor elements and take $Q = 4$ and $R = 3$. The actuator lamina is cut into 12 rectangular actuator elements. We take $K = 3$ and $L = 2$ to control the first six modes by velocity feedback control. The control gains are taken as $[G] = 0$, $h_{11} = 1.0$, $h_{12} = 0.4$, $h_{21} = 0.5$, $h_{22} = 0.4$, $h_{31} = 0.3$, and $h_{32} = 0.25$. Active control begins at $t = 0.1$ s; the response of the midpoint of the plate is shown in Fig. 3 and the time history of each controlled mode is shown in Fig. 4. The control voltage distribution of the 12 actuator elements is shown in Fig. 5.

Note that the 12th, 21st, and 32nd modes, which are uncontrollabe when using one fully covered piezoelectric lamina with spatially uniform distribution of control voltage,[16] are now controllable due to the segmentation of the actuator lamina.

It can be seen in Fig. 3 that the vibrations of the plate are suppressed very quickly and only the uncontrolled modes remain after 0.2 s. In this case, the equivalent damping ratios for the first six modes of the controlled plate are $\zeta_{11}^c = 0.199$, $\zeta_{12}^c = 0.228$, $\zeta_{21}^c = 0.204$, $\zeta_{22}^c = 0.311$, $\zeta_{31}^c = 0.226$, and $\zeta_{32}^c = 0.281$, respectively.

The numerical simulations show that the DPE method is effective for vibration control of smart plates. When the maximum control

Table 1 Material and dimensional parameters

Material	a, mm	b, mm	h, mm	Y, Gpa	ρ, kg/m^3	μ	e_{31}, N/Vm^2
PVDF	400	300	0.1	2	1780	0.3	0.06
Aluminum	400	300	0.5	69	2700	0.3	——
PZT	400	300	0.2	63	7600	0.3	10.6

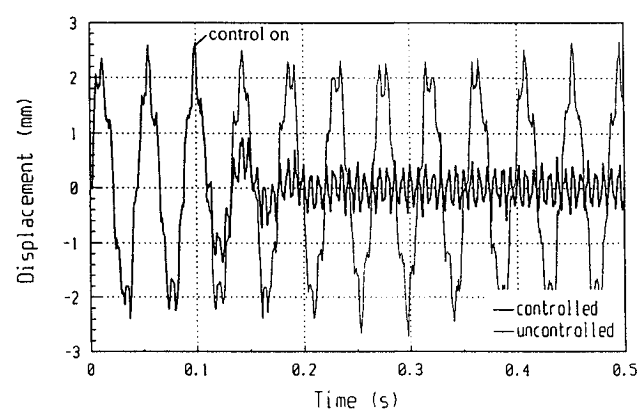

Fig. 3 Displacement response of the center of the controlled plate.

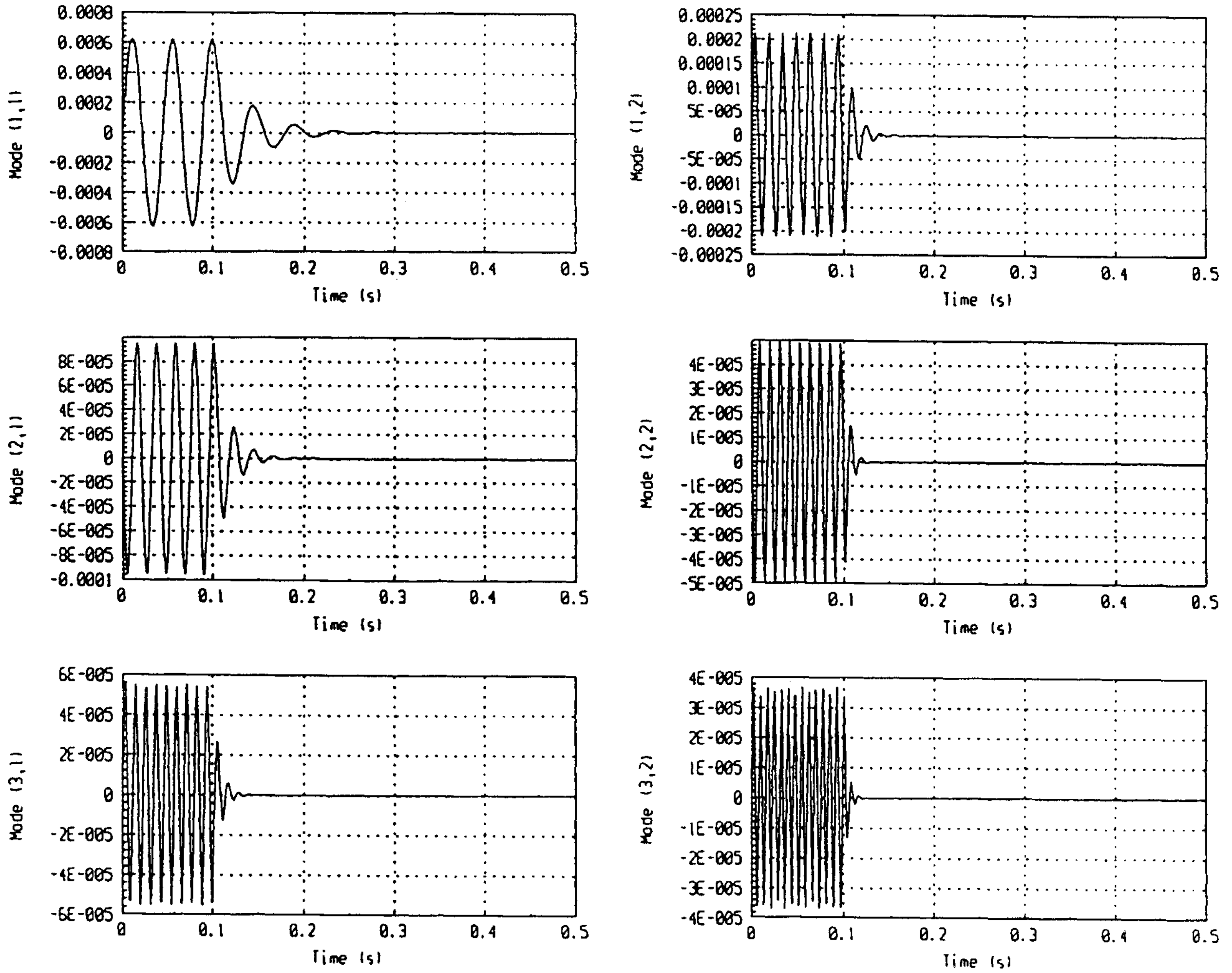

Fig. 4 Modal coordinates observed by the modal sensor.

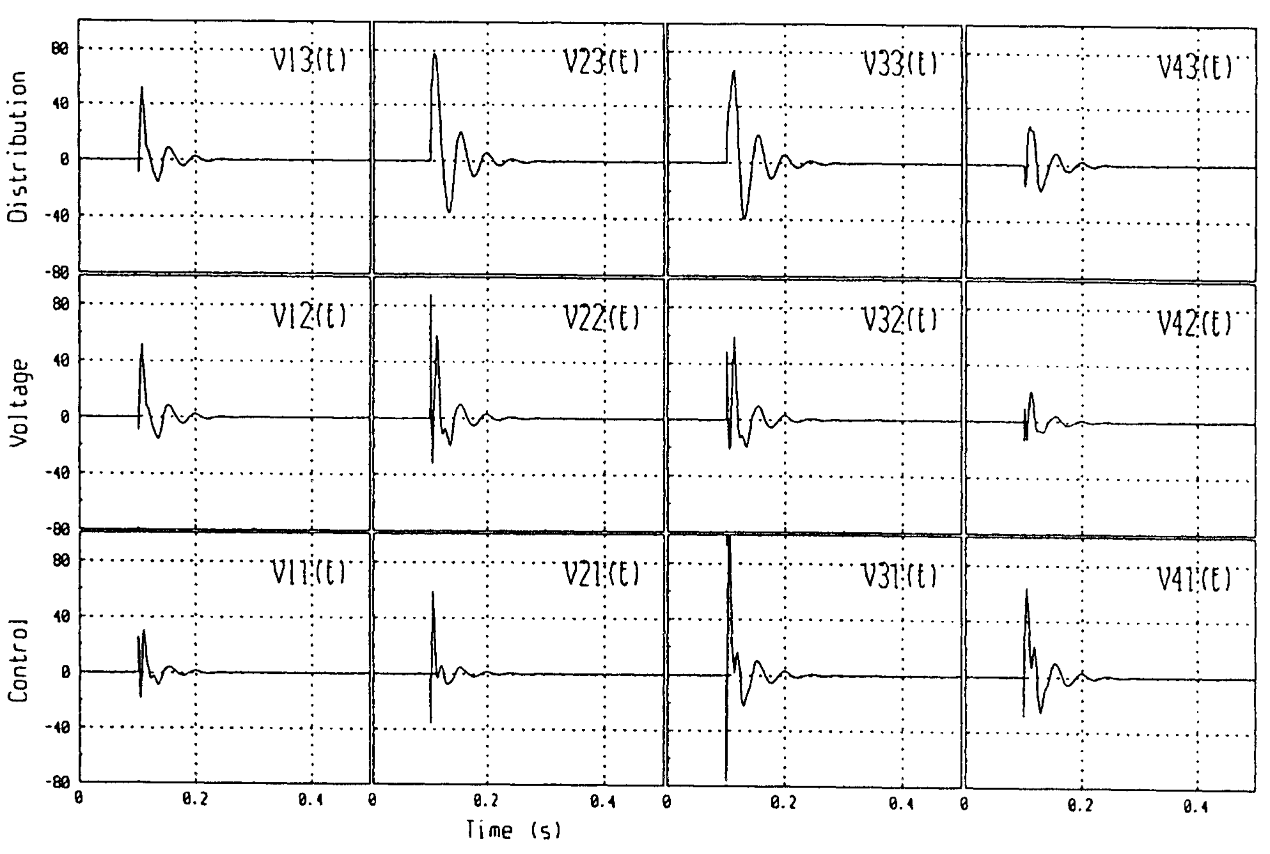

Fig. 5 Control voltage distribution.

voltage is less than 100 V, the vibrations of the plate are effectively suppressed within a short time. If the voltages applied on the actuator elements are as large as possible, the suppression time will be further shortened.

VII. Conclusion

The DPE method, a new method for active control of plates, is presented that includes methods for designing piezoelectric modal sensors as efficiently as the IMSC method. In the DPE method, the entire distributed piezoelectric sensor and actuator bonded to the plate are cut into many independent elements, the modal coordinates and modal velocities are observed from the output signals such as electric charge and electric current from the sensor elements, and the modal actuator is developed by modifying the space distribution of the applied voltages. The proper voltages applied to the actuator elements can be obtained with feedback from the observed modal coordinates and modal velocities from the sensor elements. The simulation results show that the DPE method is effective in the vibration control of smart plates. Using the DPE method, the configuration of the actuator and the sensor is not greatly changed by the changes of the number of the controlled modes and the changes of the parameters of the plate. Furthermore, the controllability and observability of the plate are improved.

Acknowledgments

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